

A Direct Physical Method for Determining the Area of a Circle Without Using Pi

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Abstract

This paper presents a physical and algebraic method for determining the exact area of a circle without using the constant pi. The method uses equal volumes of water, verified by weight, and proportional depth measurements in two containers: a cylindrical container with the target diameter, and a square or rectangular container with known base area. Once the area of one circle is measured, a simple algebraic ratio allows exact calculation of any other circle's area. The method avoids irrational numbers and produces exact values.

1. Introduction

The standard formula for circle area, " $A = \pi r^2$ ", requires the use of an irrational constant. Any numerical value for pi is necessarily an approximation, which means the resulting area is also an approximation. This paper describes a method that determines circle area exactly, without using pi, by comparing equal volumes of water in two containers.

2. Physical Principle

Water has uniform density. Equal weights of water correspond to equal volumes. If two containers hold the same weight of water, they contain the same volume. This allows direct comparison between shapes using measurable depths.

3. Measurement Setup

- 1. Select a cylindrical container with the diameter of the circle you want to measure.**
- 2. Pour a random amount of water into the cylinder and weigh the water.**
- 3. Pour the same quantity of water, measured by weight, into a square or rectangular container with known base area A_{rect} .**
4. Measure the depth d_{rect} in the square or rectangular container.
5. Measure the depth d_{circle} in the cylindrical container.

Equal water weight $\rightarrow$ equal volume $\rightarrow$ valid comparison

4. Derivation

Let:

- A_{rect} = base area of the square or rectangular container
- d_{rect} = water depth in the square/rectangular container
- A_{circle} = area of the circle
- d_{circle} = water depth in the cylindrical container

Equal volumes imply:

$$A_{rect} \cdot d_{rect} = A_{circle} \cdot d_{circle}$$

Solve for the circle's area:

$$A_{circle} = A_{rect} \cdot d_{rect} / d_{circle}$$

This value is exact. No pi is used.

5. Generalization to Any Circle

Once the area of one circle is known, any other circle's area follows from diameter scaling.

Let:

- A_1 = area of the first measured circle
- d_1 = diameter of the first circle
- d_2 = diameter of the new circle
- A_2 = area of the new circle

Circle area scales with the square of the diameter:

$$A_2 = A_1 \cdot (d_2 / d_1)^2$$

This uses only algebra and measurable diameters.

6. Discussion

This method shows that pi is not required for determining circle area. The physical measurement provides an exact value for one circle, and the algebraic ratio extends that precision to all circles. The approach avoids infinite decimals and rounding, relying only on measurable quantities and proportional relationships.

7. Conclusion

The area of a circle can be determined exactly without pi. Equal volumes of water, verified by weight, allow direct measurement of one circle's area. Diameter-squared scaling then generalizes the result to any other circle.

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